Circles and Angles

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Here are the key theorems relating circles and angles.

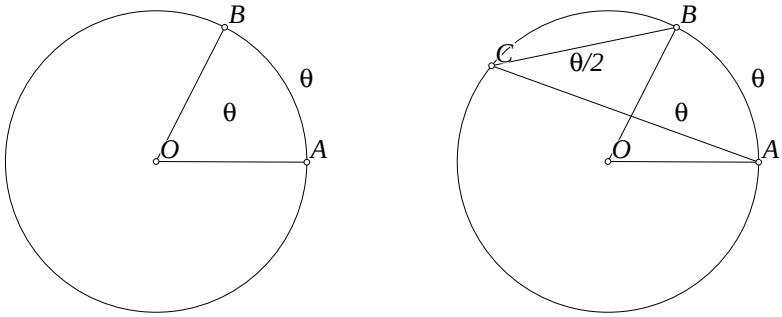

Figure 1: Central Angle and Inscribed Angle

The diagram on the left of Figure 1 gives the definiton of the measure of an arc of a circle. The arc from A to B is the same as the angle $\angle\theta$. In other words, if $\theta = 42°$, then the arc AB also has measure $42°$.

On the right of Figure 1 we see that an inscribed angle is half of the central angle. In other words, $\angle AOB = 2\angle ACB$.

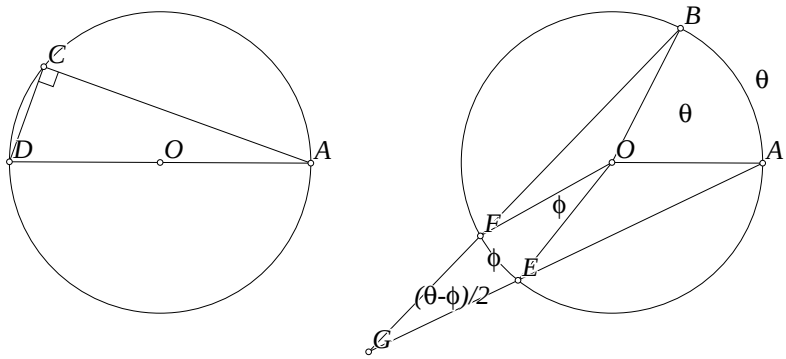

Figure 2: Right Angle, Difference of Angles

On the left of Figure 2 is illustrated the fact that an angle inscribed in a semicircle is a right angle. On the right of Figure 2 we see that an angle that cuts two different arcs of a circle has measure equal to half the difference of the arcs. In other words, $\angle AOB - \angle FOE = 2\angle EGF$.

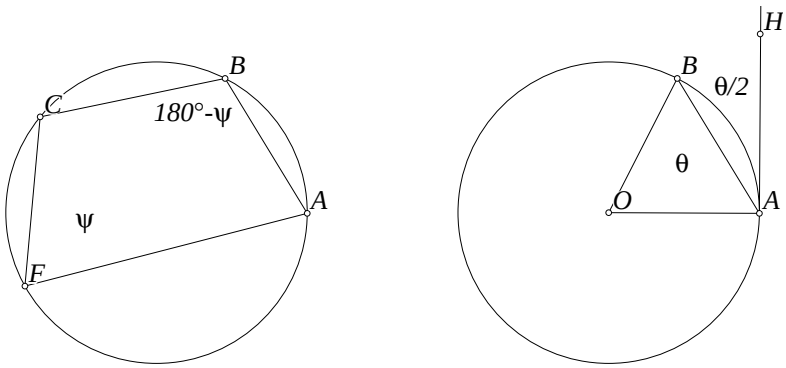

Figure 3: Inscribed Quadrilateral, Tangent

Finally, in Figure 3 we see that if a quadrilateral is inscribed in a circle, then the opposite angles add to $180°$, and conversely, if the two opposite angles of a quadrilateral add to $180°$, then the quadrilateral can be inscribed in a circle.

On the right, we see that a tangent line cuts off half of the inscribed angle: $\angle AOB = 2\angle HAB$.